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PLASTIC BUCKLING OF EXTRUDED COMPOSITE SECTIONS IN COMPRESSION

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SUMMARY

The theory of plastic buckling, originally derived for individual plates, has been extended to apply to combinations of plates such as H- and Z-sections. The theory has been applied specifically to extruded H-sections of 75S-T6 aluminum alloy; the calculated buckling stresses agree satisfactorily with the stresses obtained experimentally.

INTRODUCTION

The local buckling of aircraft section columns (H-, Z-, and channel-sections) has been studied extensively at the National Advisory Committee for Aeronautics. Experimental data representing the results of several years of work in this field are summarized in reference 1; the collection of the facts about local buckling in this reference may be considered fairly complete. In the field of theoretical studies, the situation is not so fortunate. The calculation of the local buckling stress in the elastic range has been described in reference 2; but so far no rational method has appeared in the literature for the corresponding calculation in the plastic range. With the advent of the theory of plastic buckling described in reference 3, and the subsequent experimental confirmation of this theory (reference 4), the basis has been laid for a method of computation of local buckling stresses in the plastic range. The purpose of the present work is to present such a method and to show that the results obtained will predict satisfactorily the buckling stresses of flanged section columns in the plastic range.

SYMBOLS

b	width, inches
t	thickness, inches

E	Young's modulus of elasticity, psi
E_s	secant modulus, psi
E_t	tangent modulus, psi
λ	half wave length of buckles
Λ	half wave length of buckles computed on basis of elasticity
D	plate stiffness in bending $\left(\frac{Et^3}{9}\right)$
ϵ	elastic buckling strain
σ	buckling stress, psi
n	number of outstanding flanges
C_1	plasticity coefficient $\left(\frac{1}{4} + \frac{3}{4}\frac{E_t}{E_s}\right)$
η	ratio of actual buckling stress to buckling stress computed on basis of elasticity $\left(\frac{\sigma_{pl}}{\sigma_{el}}\right)$
μ	Poisson's ratio
k	nondimensional coefficient dependent upon plate proportions and edge conditions

Subscripts:

W	web
F	flange
pl	in the plastic range
el	in the elastic range
cr	critical

METHOD OF COMPUTATION

The theory of plastic buckling described in reference 3 applies only to individual plates. A composite aircraft section, such as an H- or Z-section, on the other hand, is made up of an assembly of plates joined together, and the theory of reference 3 must be suitably modified to take into account the interaction among the plates.

For the present study, a hypothetical section which consists of a web plate with an unspecified number of flanges joined to it is considered. Evidently three conditions must be met by the theory for buckling of this composite section, whether in the elastic or in the plastic range:

(1) The critical stress of the section must be the same whether computed on the basis of a web plate restrained by flanges or on the basis of flange plates restrained by the web

(2) The half wave length of buckle must be the same in both web and flange plates (condition of continuity at joint)

(3) The sum of the stiffnesses of the individual plates must vanish at the buckling stress

If the material were elastic, these three conditions would be satisfied simultaneously by use of the moment-distribution method of reference 2. This fictitious solution is assumed to be known and may therefore be used as auxiliary information in the solution of the plastic problem.

In order to solve the problem of the buckling of a section such as the H- or Z-section in the plastic range, the first condition is met by equating the expressions in reference 3 for the plastic buckling stress of a supported plate and of a flange. The wave length of the buckles is retained in these expressions and is left indeterminate. Both sides of the equation may be divided by the corresponding expressions which would be obtained in the elastic region without violating the equality. The second condition is now met by requiring the actual half wave length on both sides of the equation to be the same; the half wave length may then be solved for in terms of the dimension ratios, the restraints, the plasticity coefficient, and the fictitious wave length which the structure would assume if the material were elastic. The third condition establishes a certain necessary relation between the restraints on the web and on the flanges. With the half wave length known, the critical stress may then be computed by a rapid trial-and-error process.

RESULTS AND DISCUSSION

The general relations from which the critical compressive stress of any composite section comprising a web with flange may be found are given in the appendix.

Table I shows certain typical basic relations independent of material, which hold for H-sections of specified thickness and width ratios. The thickness ratios used were from 0.5 to 2.0. The width ratios used were from 0.4 to 1.0. Expressions for $\left(\lambda/b_F\right)^2$ are presented for use in calculation of the plasticity factor η. This factor is an over-all plasticity coefficient defined as the ratio of the true critical stress to the critical stress calculated on the basis of perfect elasticity. Since the expressions for η and $\left(\lambda/b_F\right)^2$ contain the parameter C_1, which is a plasticity coefficient obtained from the stress-strain curve for the material, a trial-and-error method must be used to obtain the critical stress from these relations.

In order to illustrate the method, calculations have been made for extruded H-sections of 75S-T6 aluminum alloy of various dimension ratios. The calculations made by the present method are presented in figure 1(a). The H-sections selected for the calculations were assumed to be of constant thickness $\left(\frac{t_F}{t_W} = 1.0\right)$ but of different width ratios $\left(\frac{b_F}{b_W} = 0.5 \text{ and } 0.8\right)$. Stresses are plotted against the critical elastic buckling strain. The stress-strain curve used to compute the buckling curves are shown on the same scale for comparison. The experimental points were taken from reference 1 and have been slightly adjusted to a single mean stress-strain curve agreeing with that shown in the figure. The width ratios of the actual specimens differed but in general lay between the values assigned to the calculated curves which plot within a band shown by the hatched area about 1/2 ksi in width. Most of the experimental points are seen to lie within 1 ksi of the band; at the extreme end of the curve, however, the experimental points lie 2 to 4 ksi below the band.

In order to show the effect of dimension ratios better, the upper segment of figure 1(a), with additional computed curves, is plotted to a larger scale in figure 1(b). The hatched region is the same as that shown in the previous figure. For constant thickness $\left(\frac{t_W}{t_F} = 1\right)$, a range of width ratios from 0.4 to 1.0 is found to create a band only 0.6 ksi in width. Decreasing the value of t_W/t_F raises the critical stresses, since the strength of the flange plate is thus increased at the expense of the web plate. Conversely, increasing the value of t_W/t_F lowers the stresses.

CONCLUDING REMARKS

A method has been presented for determining the buckling stresses in the plastic range for flanged-section columns. Calculations by this method for extruded H-sections are compared with experimental results and the agreement is found to be satisfactory.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Air Force Base, Va., August 25, 1949

APPENDIX

DETAILS OF ANALYSIS

A composite section formed of a web plate and n outstanding flanges ($n = 2$ for Z- or channel-sections, $n = 4$ for H-sections) is considered. The calculation of the critical stress in either the elastic or the plastic region must satisfy the following conditions:

(1) The critical stress must be the same whether computed on the basis of the web or a flange plate

(2) The half wave length of the buckles must be the same in all plates because of continuity at the joints

(3) The stability criterion must be satisfied

Formulas are given in reference 3 for the critical stress $\left(\sigma_{pl}\right)_W$ of a web plate and $\left(\sigma_{pl}\right)_F$ of a flange plate in the plastic region. These formulas are, if subscripts W and F are used to denote web and flange,

$$\left(\sigma_{pl}\right)_W = \frac{E_s}{E}\left[C_1\left(\frac{b_W}{\lambda_W}\right)^2 + \left(\frac{\lambda_W}{b_W}\right)^2 f_1\left(\epsilon_W\right) + f_2\left(\epsilon_W\right)\right]\left(\frac{\pi^2 D}{b^2 t}\right)_W \tag{1}$$

$$\left(\sigma_{pl}\right)_F = \frac{E_s}{E}\,\frac{2}{\pi^2}\,\frac{C_1\left(\frac{b_F}{\lambda_F}\right)^2 + \left(\frac{\lambda_F}{b_F}\right)^2 f_3\left(\epsilon_F\right) + f_4\left(\epsilon_F\right)}{\frac{1}{3} + 0.1184\epsilon_F + 0.0107\epsilon_F^{\,2}}\left(\frac{\pi^2 D}{b^2 t}\right)_F \tag{2}$$

where

b width of plate

t thickness of plate

$D = \frac{Et^3}{9}$

ϵ elastic restraint

λ half wave length of buckles in the plastic region

$$f_1(\epsilon_W) = \frac{0.0237\epsilon_W^2 + 0.297\epsilon_W + 0.500}{0.00461\epsilon_W^2 + 0.0947\epsilon_W + 0.500}$$

$$f_2(\epsilon_W) = \frac{0.0114\epsilon_W^2 + 0.1894\epsilon_W + 1.000}{0.00461\epsilon_W^2 + 0.0947\epsilon_W + 0.500}$$

$$f_3(\epsilon_F) = \frac{0.00718\epsilon_F^2 + 0.0506\epsilon_F}{0.0530\epsilon_F^2 + 0.585\epsilon_F + 1.647}$$

$$f_4(\epsilon_F) = \frac{0.0192\epsilon_F^2 + 0.1742\epsilon_F + 0.500}{0.0530\epsilon_F^2 + 0.585\epsilon_F + 1.647}$$

$$C_1 = \frac{1}{4} + \frac{3}{4}\frac{E_t}{E_s}$$

(The functions of ϵ are close approximations, since they were found from an energy method.)

If the material were elastic

$$E_s = E_t = E$$

$$C_1 = 1$$

and

$$\left(\sigma_{el}\right)_W = \left[\left(\frac{b_W}{\Lambda_W}\right)^2 + \left(\frac{\Lambda_W}{b_W}\right)^2 f_1\left(\epsilon_W\right) + f_2\left(\epsilon_W\right)\right] \left(\frac{\pi^2 D}{b^2 t}\right)_W \tag{3}$$

$$\left(\sigma_{el}\right)_F = \frac{2}{\pi^2} \frac{\left(\frac{b_F}{\Lambda_F}\right)^2 + \left(\frac{\Lambda_F}{b_F}\right)^2 f_3\left(\epsilon_F\right) + f_4\left(\epsilon_F\right)}{\frac{1}{3} + 0.1184\epsilon_F + 0.0107\epsilon_F{}^2} \left(\frac{\pi^2 D}{b^2 t}\right)_F \tag{4}$$

where Λ is the half wave length of the buckles that would be obtained.

From the first condition to be satisfied

$$\left(\sigma_{pl}\right)_W = \left(\sigma_{pl}\right)_F$$

and

$$\left(\sigma_{el}\right)_W = \left(\sigma_{el}\right)_F$$

From these relations it follows that

$$\left(\frac{\sigma_{pl}}{\sigma_{el}}\right)_W = \left(\frac{\sigma_{pl}}{\sigma_{el}}\right)_F \tag{5}$$

When the equality is presented in this nondimensional form, the effect of taking Poisson's ratio equal to 1/2 is minimized. In the notation of reference 3, either side of equation (5) may be recognized as the over-all plasticity coefficient η.

Substitution of equations (1) to (4) in equation (5) yields

$$\eta = \frac{E_s}{E}\,\frac{C_1\left(\frac{b_W}{\lambda_W}\right)^2 + \left(\frac{\lambda_W}{b_W}\right)^2 f_1(\epsilon_W) + f_2(\epsilon_W)}{\left(\frac{b_W}{\Lambda_W}\right)^2 + \left(\frac{\Lambda_W}{b_W}\right)^2 f_1(\epsilon_W) + f_2(\epsilon_W)}$$

$$= \frac{E_s}{E}\,\frac{C_1\left(\frac{b_F}{\lambda_F}\right)^2 + \left(\frac{\lambda_F}{b_F}\right)^2 f_3(\epsilon_F) + f_4(\epsilon_F)}{\left(\frac{b_F}{\Lambda_F}\right)^2 + \left(\frac{\Lambda_F}{b_F}\right)^2 f_3(\epsilon_F) + f_4(\epsilon_F)} \tag{6}$$

From the second condition, which requires continuity at the joints,

$$\Lambda_W = \Lambda_F = \Lambda$$

and (7)

$$\lambda_W = \lambda_F = \lambda$$

so that equation (6) may be written

$$\eta = \frac{E_s}{E}\,\frac{C_1\left(\frac{b_W}{\lambda}\right)^2 + \left(\frac{\lambda}{b_W}\right)^2 f_1(\epsilon_W) + f_2(\epsilon_W)}{\left(\frac{b_W}{\Lambda}\right)^2 + \left(\frac{\Lambda}{b_W}\right)^2 f_1(\epsilon_W) + f_2(\epsilon_W)}$$

$$= \frac{E_s}{E}\,\frac{C_1\left(\frac{b_F}{\lambda}\right)^2 + \left(\frac{\lambda}{b_F}\right)^2 f_3(\epsilon_F) + f_4(\epsilon_F)}{\left(\frac{b_F}{\Lambda}\right)^2 + \left(\frac{\Lambda}{b_F}\right)^2 f_3(\epsilon_F) + f_4(\epsilon_F)} \tag{8}$$

Solution of equation (8) for λ yields a relation of the form

$$\left(\frac{\lambda}{b_F}\right)^2 = \frac{-B \pm \sqrt{B^2 - 4AFC_1}}{2A} \tag{9}$$

where

$$A = \left[1 + \left(\frac{\Lambda}{b_F}\right)^2 f_4(\epsilon_F)\right]\left(\frac{b_F}{b_W}\right)^2 f_1(\epsilon_W) - \left[\left(\frac{b_W}{b_F}\right)^2 + \left(\frac{\Lambda}{b_F}\right)^2 f_2(\epsilon_W)\right] f_3(\epsilon_F)$$

$$B = \left[1 + \left(\frac{\Lambda}{b_F}\right)^2 f_3(\epsilon_F)\right] f_2(\epsilon_W) - \left[\left(\frac{b_W}{b_F}\right)^2 + \left(\frac{\Lambda}{b_F}\right)^2\left(\frac{b_F}{b_W}\right)^2 f_1(\epsilon_W)\right] f_4(\epsilon_F)$$

$$F = \left(\frac{\Lambda}{b_F}\right)^2 \left\{\left[\left(\frac{\Lambda}{b_F}\right)^2 f_3(\epsilon_F) + f_4(\epsilon_F)\right]\left(\frac{b_W}{b_F}\right)^2 - \left[\left(\frac{\Lambda}{b_F}\right)^2\left(\frac{b_F}{b_W}\right)^2 f_1(\epsilon_W) + f_2(\epsilon_W)\right]\right\}$$

The third condition expresses the stability criterion, in the notation of reference 2:

$$S^{IV} + \frac{nS^{III}}{2} = 0 \tag{10}$$

or, in the restraint notation of this paper

$$\epsilon_W = - \frac{\frac{n}{2}}{\left(\frac{t_W}{t_F}\right)^3\left(\frac{b_F}{b_W}\right)} \epsilon_F \tag{11}$$

The restraint ϵ_W on the web is thus known in terms of the restraint ϵ_F on the flange and vice versa; the restraints, as well as Λ, are found from the solution obtained on the basis of a perfectly elastic material.

The sole remaining unknown quantity on the right-hand side of equation (9) is the plasticity coefficient C_1, which is a function of stress. Relations of the type shown in table I, which hold for the dimension ratios shown, independently of the material, are therefore obtained.

Upon the assumption of a critical stress as a first approximation, λ is determined from equation (9); either of the expressions (8) for η, whichever is more convenient, may now be computed and the critical stress found, to see if it agrees with the assumed first approximation. If not, the trial-and-error procedure is followed in the usual way. Convergence is especially rapid if the stress given by the stress-strain curve itself at the appropriate strain is taken as the first approximation.

REFERENCES

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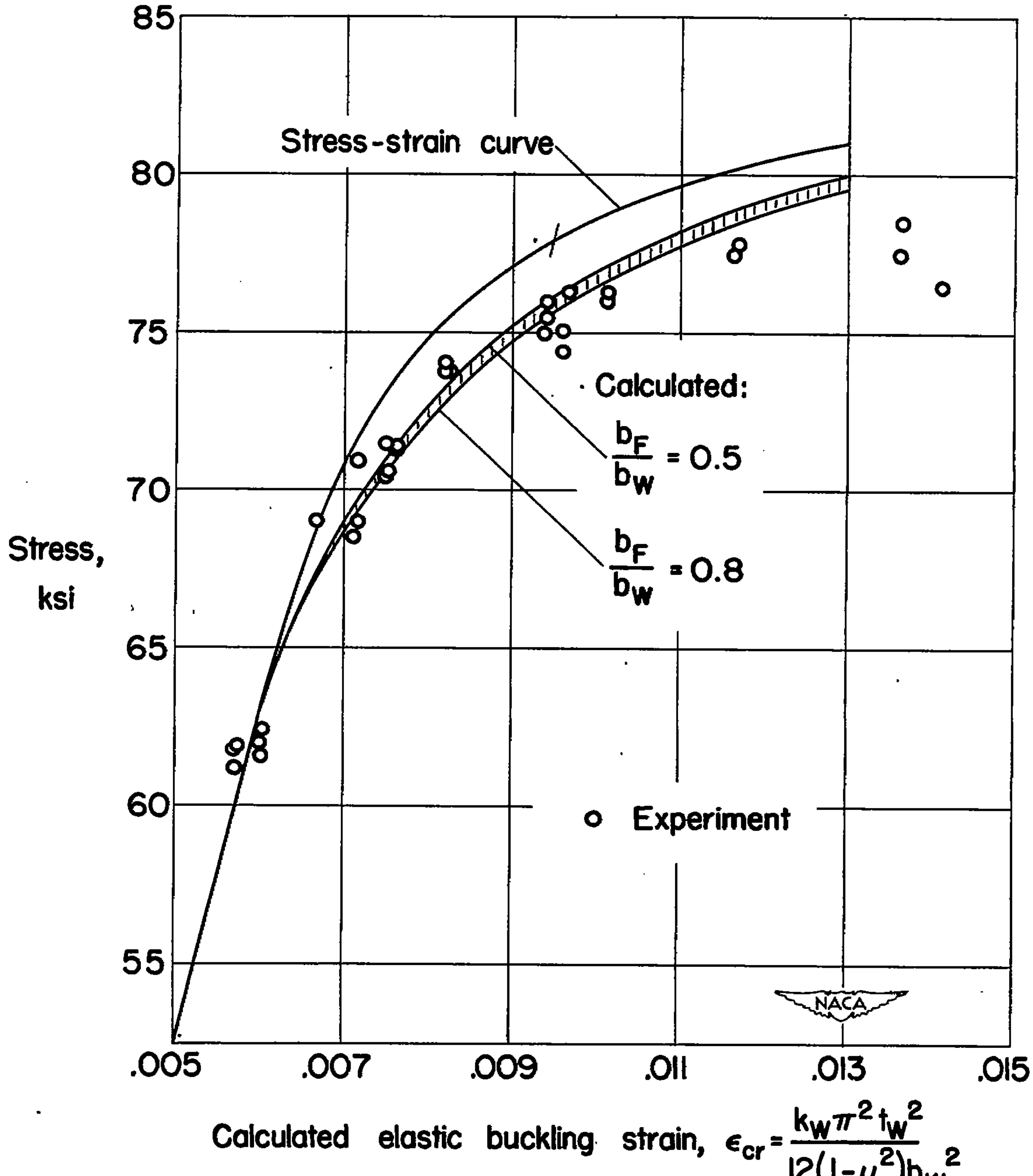
TABLE I

GENERAL EXPRESSIONS FOR THE PLASTICITY REDUCTION FACTOR η WHICH

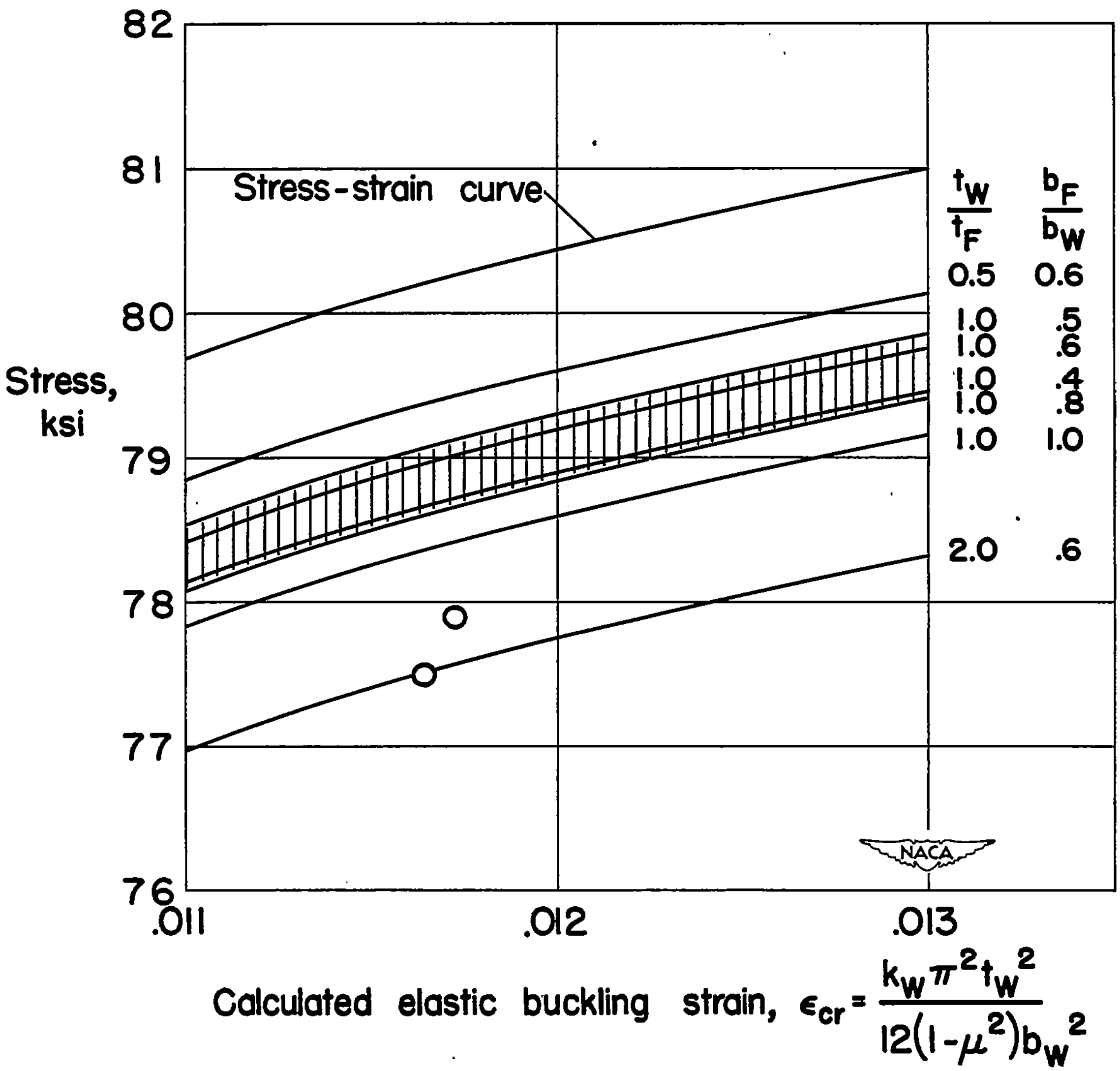
APPLY TO H-SECTIONS WITH CERTAIN DIMENSIONAL RATIOS

$\frac{t_W}{t_F}$	$\frac{b_F}{b_W}$	$\left(\frac{\lambda}{b_F}\right)^2$	η
1.0	0.4	$3.52 + 5.31\sqrt{0.442 + C_1}$	$\frac{E_s}{E}\left[2.234\left(\frac{b_F}{\lambda}\right)^2 C_1 + 0.00996\left(\frac{\lambda}{b_F}\right)^2 + 0.677\right]$
1.0	.5	$5.38 + 3.98\sqrt{1.824 - C_1}$	$\frac{E_s}{E}\left[1.870\left(\frac{b_F}{\lambda}\right)^2 C_1 + 0.02506\left(\frac{\lambda}{b_F}\right)^2 + 0.566\right]$
1.0	.6	$4.62 + 2.90\sqrt{2.53 - C_1}$	$\frac{E_s}{E}\left[1.758\left(\frac{b_F}{\lambda}\right)^2 C_1 + 0.0308\left(\frac{\lambda}{b_F}\right)^2 + 0.533\right]$
1.0	.8	$3.47 + 1.145\sqrt{9.20 + C_1}$	$\frac{E_s}{E}\left[1.604\left(\frac{b_F}{\lambda}\right)^2 C_1 + 0.0406\left(\frac{\lambda}{b_F}\right)^2 + 0.485\right]$
1.0	1.0	$2.95 + 0.827\sqrt{12.72 + C_1}$	$\frac{E_s}{E}\left[1.540\left(\frac{b_F}{\lambda}\right)^2 C_1 + 0.0458\left(\frac{\lambda}{b_F}\right)^2 + 0.468\right]$
.5	.6	$8.18 + 8.77\sqrt{0.870 + C_1}$	$\frac{E_s}{E}\left[2.65\left(\frac{b_F}{\lambda}\right)^2 C_1 + 0.00323\left(\frac{\lambda}{b_F}\right)^2 + 0.804\right]$
2.0	.6	$1.274 + 1.89\sqrt{0.452 + C_1}$	$\frac{E_s}{E}\left[1.059\left(\frac{b_F}{\lambda}\right)^2 C_1 + 0.1006\left(\frac{\lambda}{b_F}\right)^2 + 0.344\right]$



(a) $\frac{t_F}{t_W} = 1.0$; $\frac{b_F}{b_W} = 0.5$ and 0.8.

Figure 1.— Correlation of buckling curves and experimental results for extruded H-section columns of 75S-T aluminum alloy of constant thickness which fail by local instability.



(b) Enlargement of part of figure 1(a) for purpose of showing effect of additional dimension ratios.

Figure 1.– Concluded.